# Exploring our Embodied Knowing of the Gauss-Bonnet Theorem: Barn-Raising an Endo-Pentakis-Icosi-Dodecahedron

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When applied to the polyhedral[23] case, the Gauss-Bonnet Theorem determines a property of the vertices of the surface of a solid: if we add together the angles which have been taken away from 360° at each vertex (the angle deficit), the result will equal a constant 720° (Alexandrov & Zagaller, 1976). This result is surprising for the uninitiated, in that it is true of any polyhedron. For the case of the cube, it is easy to visualise: at each of its eight corners, 90° was removed in order to close it, giving a total of 720°.

In the planned workshop, we will explore the case of a polyhedron with no right angles, the endo-pentakis-icosi-dodecahedron (Cundy & Rollet, 1961; Conway, 1999), a polyhedron with 80 equilateral triangular faces. This will allow the participants to reflect on their embodied knowledge of polyhedra and the angle deficit property. Using 1-metre-edge-length faces, we will construct the polyhedron with the aid of a net which was developed based on the angle deficit idea (Knoll & Morgan, 1999).

The scale of the project will give the participants an experience of collaborative mathematics practice through the barn-raising (Knoll & Morgan, 1999; Hart, 2004). In addition, they will have an embodied experience of polyhedral geometry: they will be able to pace the area of the flat net and to physically enter the space defined by the polyhedron, allowing them a sense of the total angle deficit. This last experience will help to initiate reflections on the relationship between our understanding of space and our motor-control system (Lakoff & Núñez, 2000).

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[23] We are defining polyhedra as having genus 0.